Topology Optimization for Electromagnetic Problems

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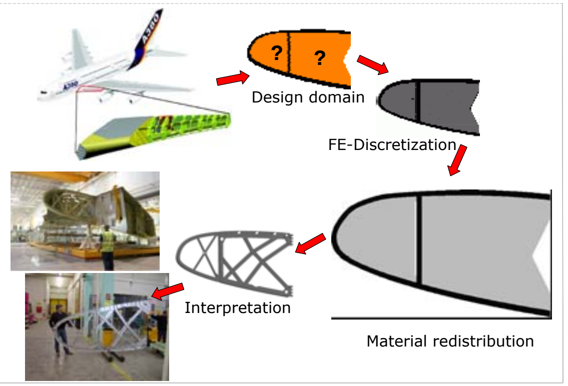
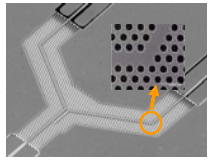
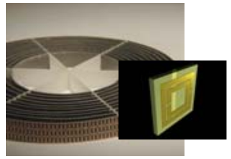
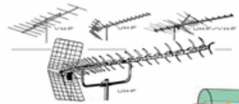
The topology optimization method

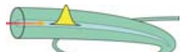


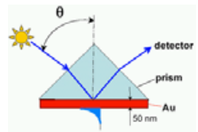
Design problems in electromagnetics


Photonic crystal waveguides


Metamaterials and cloaking


Antennas


Passive signal processing



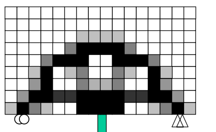
Surface plasmon polaritons

Topology optimization methods

- Homogenization approach
- Density approach (SIMP)
- Evolutionary methods
- Level-set methods
- Phase-field approaches
- Topological gradients
- etc.

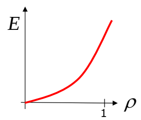
Topology optimization (SIMP-approach)

Bendsøe (1989), Zhou and Rozvany (1991), Mlejnek (1992)



$$\min_{\boldsymbol{\rho}} : \quad \Phi(\boldsymbol{\rho})$$
$$s.t. : \quad \sum_{e=1}^{N} v_e \rho_e = \mathbf{v}^T \boldsymbol{\rho} \le V^*$$
$$: \quad g_i(\boldsymbol{\rho}) \le g_i^*, \quad i = 1, ..., M$$
$$: \quad 0 < \rho_{\min} \le \boldsymbol{\rho} \le 1$$
$$: \quad \mathbf{K}(\boldsymbol{\rho})\mathbf{U} = \mathbf{F}$$

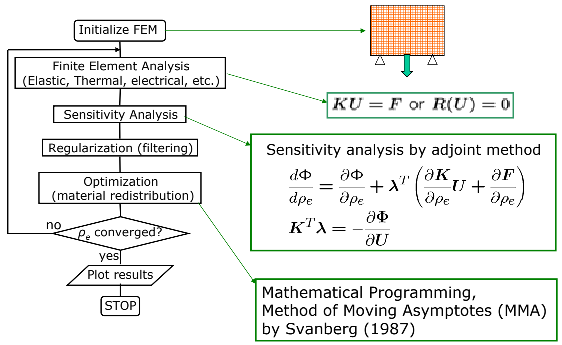
Stiffness interpolation:



$$E(\rho_e) = \rho_e^p E_0$$
$$p > 1$$

SIMP physically validated in Bendsøe and Sigmund (1998)

The topology optimization process

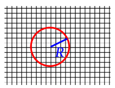


Filtering methods

Neighborhood:

$$N_e = \{ i \mid ||\mathbf{x}_i - \mathbf{x}_e|| \le R \},$$

where *R* is the filter radius.



Sensitivity filtering, Sigmund (1994,1997)

$$\widetilde{\frac{\partial c}{\partial \rho_e}} = \frac{\sum_{i \in N_e} w(\mathbf{x}_i) \rho_i \frac{\partial c}{\partial \rho_i}}{\rho_e \sum_{i \in N_e} w(\mathbf{x}_i)}$$

Density filtering, Bruns and Tortorelli (2001) and Bourdin (2001)

$$E_e(\boldsymbol{\rho}) = \tilde{\rho}_e^p E_0, \qquad \tilde{\rho}_e = \frac{\sum_{i \in N_e} w(\mathbf{x}_i) A_i \rho_i}{\sum_{i \in N_e} w(\mathbf{x}_i) A_i}$$

Black and white filters

Guest et al. (2004)
Sigmund (2007/2009)

Image morphology filters:

Erode: $\rho_e^d = \max_{i \in N_e}(\rho_i)$

Dilate: $\rho_e^e = \min_{i \in N_e}(\rho_i)$

Differentiable versions:

Erode: $\rho_e^e = e^{-\beta(1-\tilde{\rho}_e)} - (1 - \tilde{\rho}_e) e^{-\beta}$

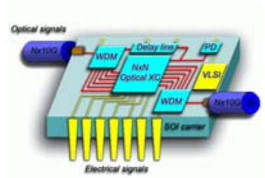
Dilate: $\rho_e^d = 1 - e^{-\beta \tilde{\rho}_e} + \tilde{\rho}_e e^{-\beta}$


No filter -> Checkerboards

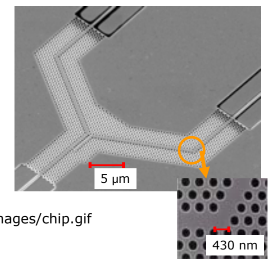

Standard filters


Morpholgy filters

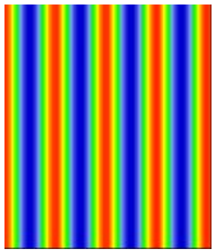
Photonic crystal based waveguides

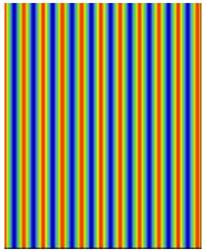


From www.research.ibm.com/photonics/images/chip.gif

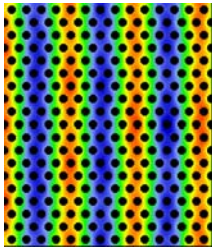


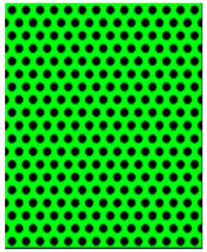
Wave propagation in homogeneous medium


Low frequency

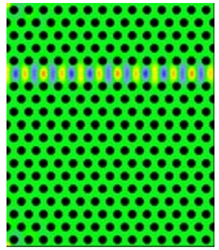

High frequency

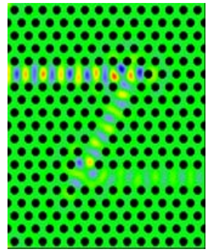
Wave propagation in periodic medium


Low frequency

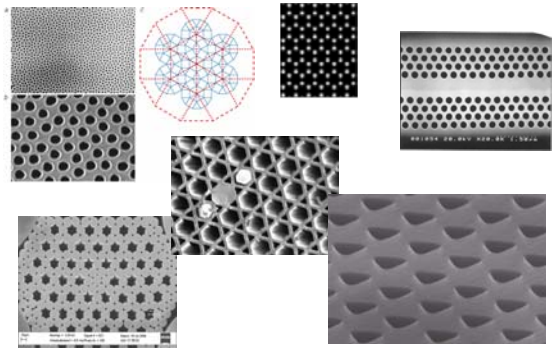

High frequency

Wave propagation in periodic medium w. defects


Line defect


Z-shaped defect

What microstructure maximizes the band gap?



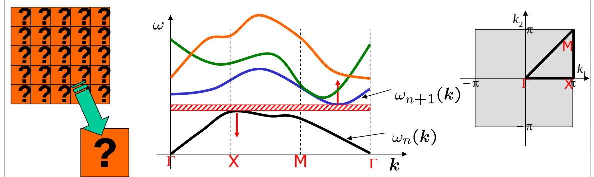
The Helmholtz equation

$$\nabla \cdot (A \nabla u) + \omega^2 B u = 0$$

$$u(\boldsymbol{x}) = U(\boldsymbol{x}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}, \quad U(\boldsymbol{x}) = U(\boldsymbol{x} + \boldsymbol{r})$$

Light propagation (TE):	$A = \frac{1}{\varepsilon_r}, \; B = \frac{\mu_r}{c^2}, \; u = H_z$
Light propagation (TM):	$A = \frac{1}{\mu_r}, \; B = \frac{\varepsilon_r}{c^2}, \; u = E_z$
Acoustics:	$A = \frac{1}{\rho}, \; B = \frac{1}{\kappa}, \; u = p$
Elastic waves:	$A = \mu, \; B = \rho$

Band gap optimization



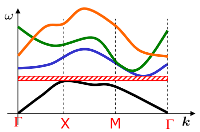
Maximize relative band gap between bands *n* and *n+1*:

$$\frac{\Delta\omega_n}{\omega_n^0} = 2\frac{\min_{\boldsymbol{k}} : \omega_{n+1}(\boldsymbol{k},\boldsymbol{\rho}) - \max_{\boldsymbol{k}} : \omega_n(\boldsymbol{k},\boldsymbol{\rho})}{\min_{\boldsymbol{k}} : \omega_{n+1}(\boldsymbol{k},\boldsymbol{\rho}) + \max_{\boldsymbol{k}} : \omega_n(\boldsymbol{k},\boldsymbol{\rho})}$$

Band gap optimization

$$\max_{\boldsymbol{\rho}\in[0,1]^N} : 2\frac{\min_{\boldsymbol{k}} : \omega_{n+1}(\boldsymbol{k},\boldsymbol{\rho}) - \max_{\boldsymbol{k}} : \omega_n(\boldsymbol{k},\boldsymbol{\rho})}{\min_{\boldsymbol{k}} : \omega_{n+1}(\boldsymbol{k},\boldsymbol{\rho}) + \max_{\boldsymbol{k}} : \omega_n(\boldsymbol{k},\boldsymbol{\rho})}$$

$$\epsilon_r(\rho_e) = 1 + \rho_e(\epsilon_{r,d} - 1)$$



Complications ...

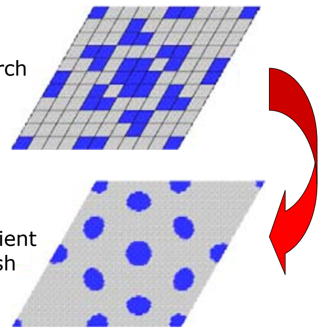
- Several eigenvalue analyses pr. iteration
- Multiple eigenvalues – complicated sensitiity analysis
- Active set strategy
- Disconnected solutions sets

NB! No penalization and regularization needed here

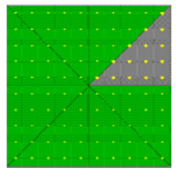
Two-step approach

Make exhaustive search on **coarse mesh**

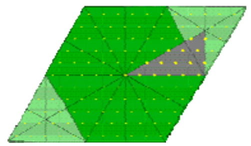
Fine design by gradient approach on fine mesh



Coarse discretization



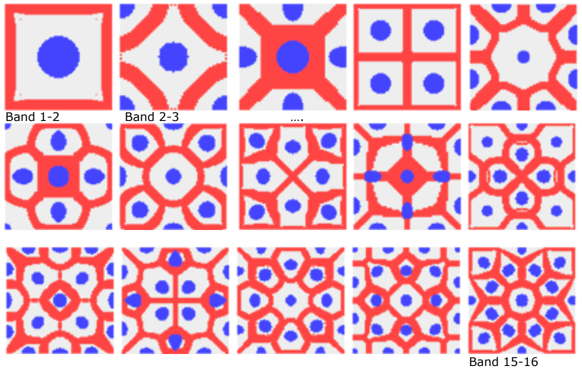
Square cell:
10 by 10 elements
15 variables
2^{15} =33768 combinations
6298 combinations after reduction



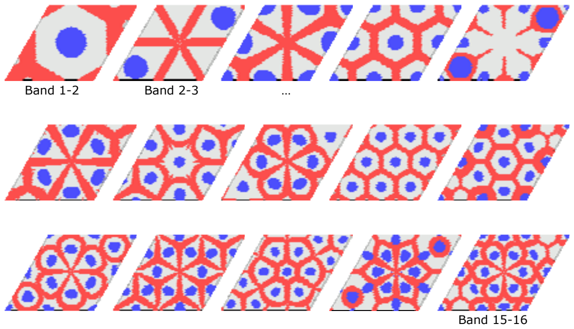
Rhombic cell:
11 by 11 elements
16 variables
2^{16} =65536 combinations
15450 combinations after reduction

Reduction: f<10%, f>90%, isolated elements, translational copies

Composite TE (red) and TM (blue) structures

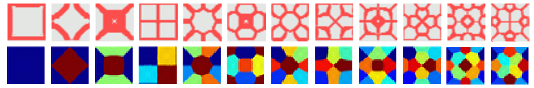


Composite TE (red) and TM (blue) structures



Geometrical quality measures

- Mesh quality factor (not too good)
- Air filling fraction (pretty good but) not good for design
- Perimeter of cell walls (for TE walls, pretty good)
- Packing factors
- Cybulski et al., "Minimization of the Renyi entropy production in the space-partitioning process", Phys Rev. E, **71**, 046130 (2005)



Conjecture

Optimal photonic band gap structures
for gaps between band n *and* n+1
can be found by a purely geometric rule:

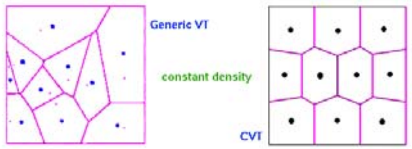
TM-polarization:
n elliptic rods with centers defined by the generators of the optimal *centroidal Voronoi tesselation*

TE-polarization:
The walls of above tesselation

Centroidal Voronoi tesselation

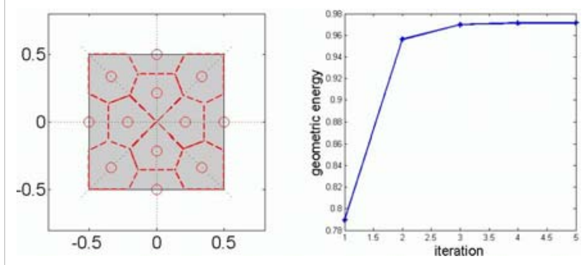
A Voronoi tessellation is called centroidal when the generating point of each Voronoi cell is also its mean (center of mass).

It can be viewed as an optimal partition corresponding to an optimal distribution of generators

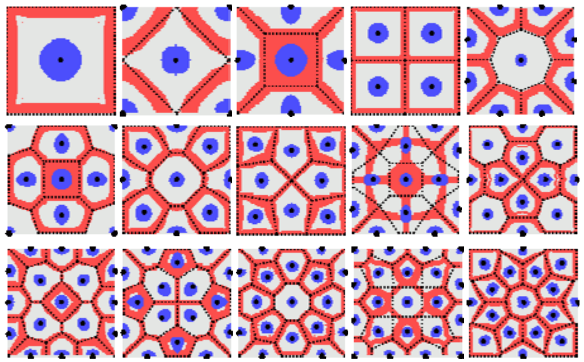


Lloyd's algorithm

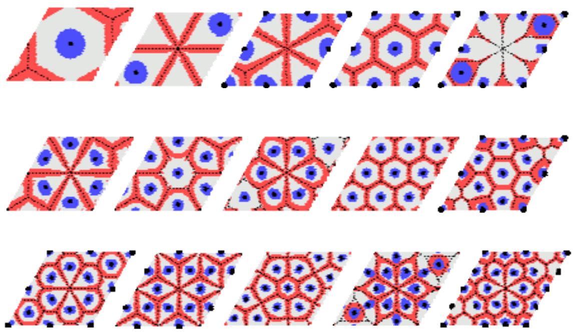
Example: *n=10*



Composite TE and TM structures



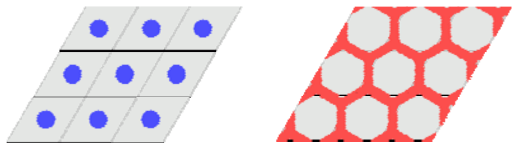
Composite TE and TM structures



"Geometric properties of optimal photonic band gap structures"

Sigmund & Hougaard, *PRL*, **2008**, *100*, 153904

The overall optimal planar photonic band gap structures are:



Comparisons to literature

"Relaxation of symmetry increases band gap"

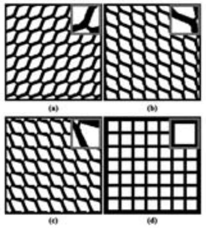
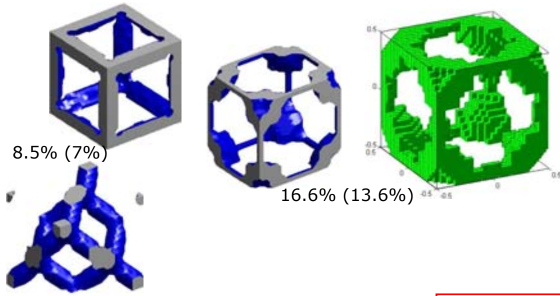


FIG. 1. Photonic crystal and unit cell (insets). (a–c) The best three photonic crystals created by the evolutionary algorithm with the following band gaps: (a) 0.3189, (b) 0.3153, (c) 0.3115. (d) Best human designed photonic crystal with a band gap of 0.2835.

Preble, Lipson and Lipson,
APL, 86, 61111 (2005)

What about 3d?

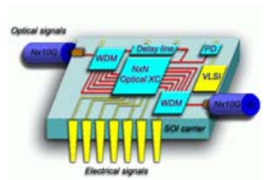


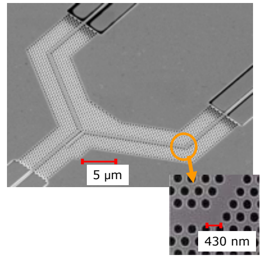
32.2% (31.4%)

By Niels Aage

Issues: Discretization, CPU-time, parallellization, symmetries

Photonic crystal based waveguides





From www.research.ibm.com/photonics/images/chip.gif

Harmonic problem

Helmholtz equation for planar waves

$$\nabla \cdot (A \nabla u) + \omega^2 B u = 0$$

plus BC's like incident waves, PML's, etc.

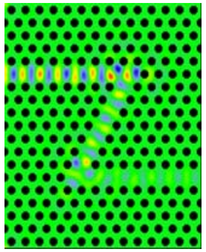
Finite Element Frequency Domain formulation

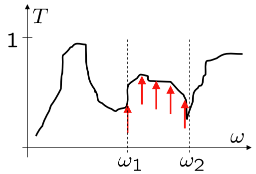
$$\left[\mathbf{K}(\rho) + i\omega \mathbf{C}(\rho) - \omega^2 \mathbf{M}(\rho)\right] \mathbf{U} = \mathbf{F}(\omega)$$

or

$$\mathbf{S}(\omega, \boldsymbol{\rho})\mathbf{U} = \mathbf{F}(\omega)$$

Objective function





$$\max_{\boldsymbol{\rho}\in[0,1]^N} \min_{\omega\in[\omega_1,\omega_2]} T(\omega, \boldsymbol{\rho})$$

The topology optimization problem

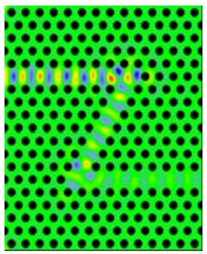
$$\begin{aligned} \max_{\boldsymbol{\rho}} \min_{k} \quad &: \quad \Phi_k(\boldsymbol{\rho}) = \frac{\omega_k}{2}\mathbf{n}^T \int_{\Gamma_{out}} A\, \Re(iu_k \nabla \bar{u}_k)\, d\Gamma \\ s.t. \quad &: \quad \mathbf{S}(\omega_k, \boldsymbol{\rho})\mathbf{U} = \mathbf{F}(\omega_k), \quad k = 1, ..., M \\ &: \quad 0 \le \boldsymbol{\rho} \le 1 \end{aligned}$$

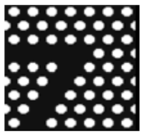
- Analytical sensitivities by the adjoint method

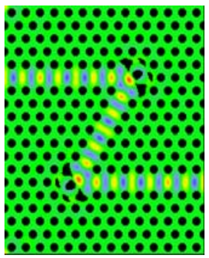
$$\frac{\partial \Phi}{\partial \rho_e} = 2\,\Re\left(\boldsymbol{\lambda}^T \frac{\partial \mathbf{S}}{\partial \rho_e}\mathbf{U} - \boldsymbol{\lambda}^T \frac{\partial \mathbf{F}}{\partial \rho_e}\right), \quad \text{where } \mathbf{S}\,\boldsymbol{\lambda} = -\frac{\partial \Phi}{\partial \rho_e}$$

- Optimization frequencies $\omega_k \in [\omega_1, \omega_2]$ updated based on Padé approximations

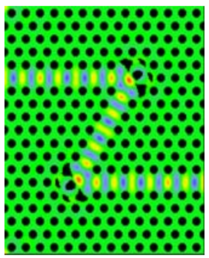
Topology Optimization of the Z-bend

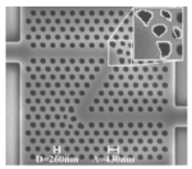


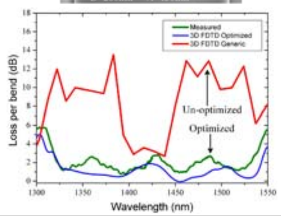




Realization and tests

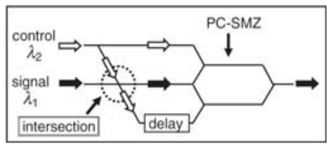


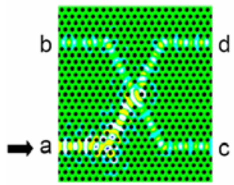


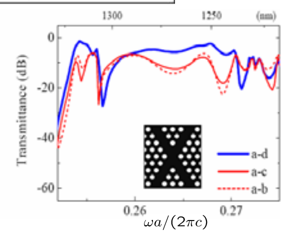


Borel et al., OE, 12(9), (2004)

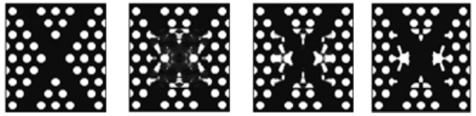
Mach-Zehnder type all-optical switch



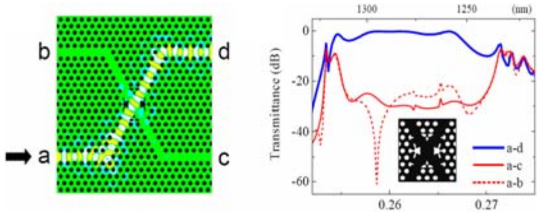




Mach-Zehnder type all-optical switch

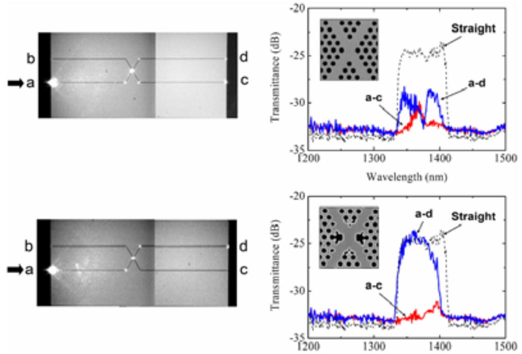


Iteration history



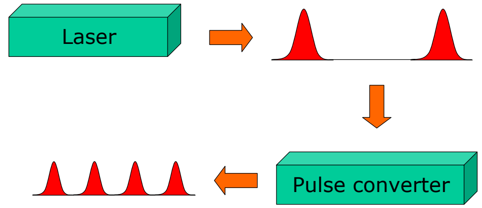
Ikeda et al., Electron Lett, 42(18), pp. 1031-1033 (2006)

Mach-Zehnder type all-optical switch



Ikeda et al., Electron Lett, 42(18), pp. 1031-1033 (2006)

Transient problems



Sensitivity analysis for time-domain simulations

Forward analysis

$$\mathbf{K}\mathbf{U}_t + \mathbf{M}\ddot{\mathbf{U}}_t = \mathbf{F}_t, \quad t = 0\ldots T$$

Objective function

$$\Phi = \int_0^T \phi(\boldsymbol{U},\boldsymbol{\rho},t)dt$$

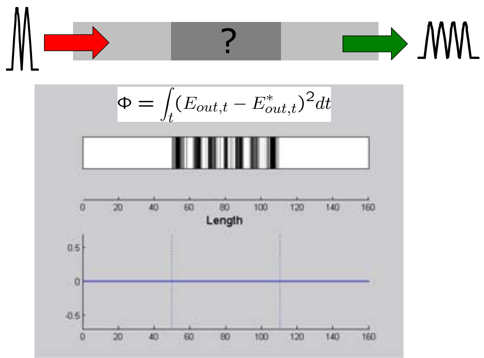
Sensitivity of objective function

$$\frac{d\Phi}{d\rho^e} = \int_0^T \left[\frac{\partial\phi_t}{\partial\rho^e} + \boldsymbol{\lambda}_t^T\left(\frac{\partial \boldsymbol{F}_t}{\partial\rho^e} - \frac{\partial\boldsymbol{K}}{\partial\rho^e}\boldsymbol{U}_t - \frac{\partial\boldsymbol{M}}{\partial\rho^e}\ddot{\boldsymbol{U}}_t\right)\right]dt$$

Adjoint analysis equation

$$\mathbf{K}\boldsymbol{\lambda}_t + \mathbf{M}\ddot{\boldsymbol{\lambda}}_t = \left(\frac{\partial\phi_t}{\mathbf{U}}\right)^T, \; t = T\ldots 0,\; \boldsymbol{\lambda}_{t=T}=0,\; \ddot{\boldsymbol{\lambda}}_{t=T}=0$$

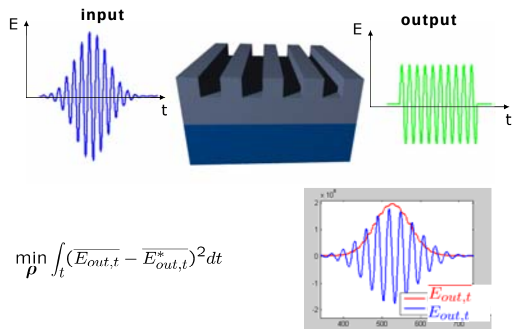
Tailoring transient response



$$\Phi = \int_t (E_{out,t} - E^*_{out,t})^2 dt$$

Dahl, Jensen and Sigmund (2008)

Pulse shaping (Lirong Yang)



$$\min_{\boldsymbol{\rho}} \int_t (\overline{E_{out,t}} - \overline{E^*_{out,t}})^2 dt$$

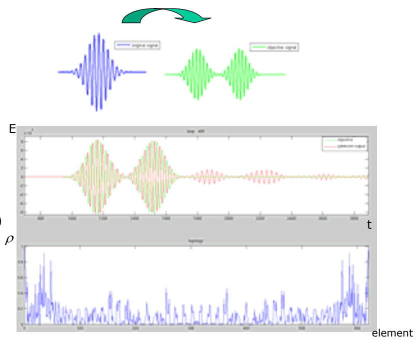
Application: Pulse Splitter

Optimized pulses (red)

vs.

Objective pulses (green)

Optimized Topology



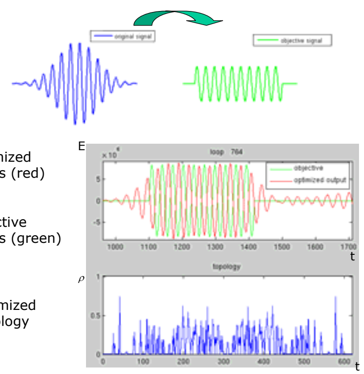
Application: Square Pulse Shaping

Optimized pulses (red)

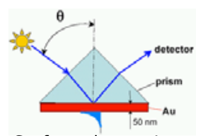
vs.

Objective pulses (green)

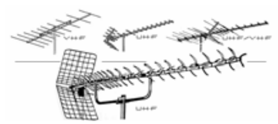
Optimized Topology



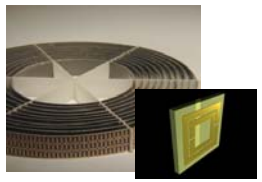
Metallic structures



Surface plasmonics

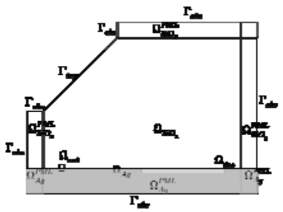
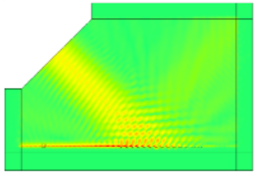


Antennas



Meta materials

Design of grating couplers (Jacob Andkjær)

$$\epsilon_r(\rho^e) = (1-\rho^e)\epsilon_{r,SiO_2} + \rho^e \epsilon_{r,Ag}$$

$\epsilon_{r,SiO_2} = 2.25$ and $\epsilon_{r,Ag} = -7.06 - 0.27i$

Results

	Fig.	PML	Initial design	iter.	Vol. constr.	Vol. frac.	Efficiency
Initial guess: GC from [2]	2a	Koshiba	-	1	-	0.264	49.7%
Example 1	2c	Koshiba	GC	371	0.5	0.234	66.1%
Initial guess: Wave design	2b	Koshiba	-	1	-	0.5	0.1%
Example 2	2d	Koshiba	Wave	367	0.5	0.293	61.6%

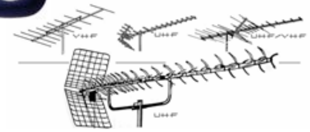
Table 1: Results for optimized designs given in Fig. 2.

Fig. 2: Initial and optimized GC designs.

Antenna design







Small antennas for hearing aids



Maxwell's equations

Maxwell's equation for harmonic waves

$$\nabla \times \left(\frac{1}{\mu_r}\nabla \times \mathbf{E}\right) + \frac{\omega^2}{c^2}\left(\epsilon_r - i\frac{\sigma}{\omega\epsilon_0}\right)\mathbf{E} = 0$$

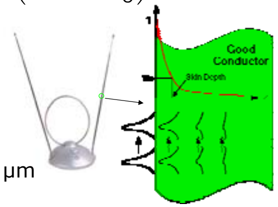
Skin depth

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

Skin depth in Copper, δ<1 μm

Domain size ~ 1 cm^3

FE-discretization ~ 20.000^3

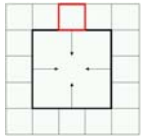


Resolving the skin depth issue

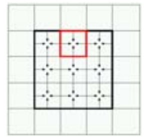
Impedance BC

$$\boldsymbol{n} \times \left(\frac{1}{\mu_r}\nabla \times \mathbf{E}\right) - i\frac{\omega}{c}f\sqrt{\left(\epsilon_r - i\frac{\sigma}{\omega\epsilon_0}\right)} \times (\mathbf{E} \times \boldsymbol{n}) = 0$$

Cut through square conductor



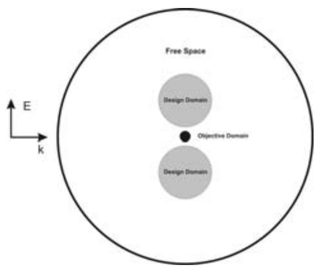
Conventional discretization



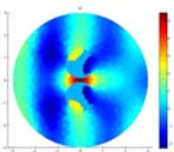
TopOpt discretization

Aage, Mortensen and Sigmund (2009), in preparation

2D test case, field concentrator



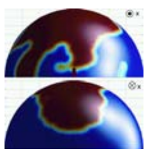
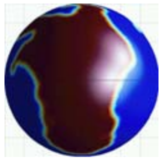
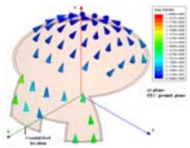
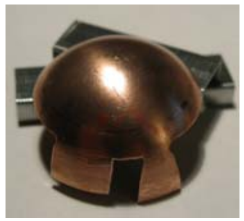
ρ



H_z

Aage, Mortensen and Sigmund (2009), in preparation

Small antennas for hearing aids

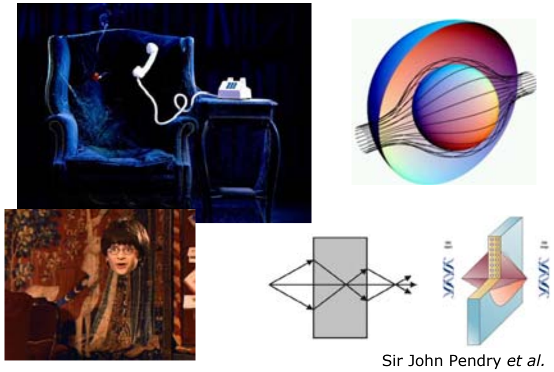





Erentok and Sigmund (2009), submitted

Meta-materials

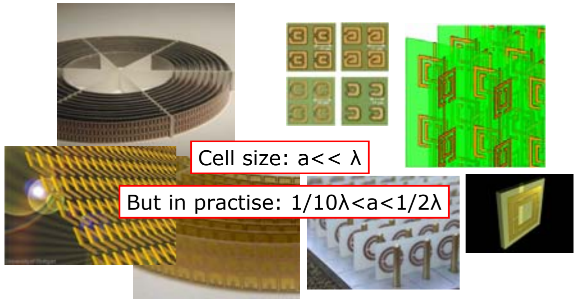
Positive refractive index

Cloaking and perfect lensing



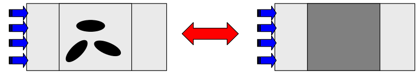
Sir John Pendry *et al.*

Electromagnetic metamaterials



Microwave regime: a ~ 1cm
Optical regime: a ~ 100nm

Parameter retrieval from S-paramters



$$n^H = \frac{1}{kd}\left(\cos^{-1}\left[\frac{1}{2S_{21}}(1-S_{11}S_{22}+S_{21}^2)\right]+2p\pi\right)$$

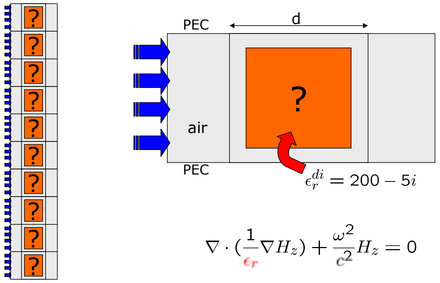
$$z^H = \sqrt{\frac{(1+S_{11})(1+S_{22})-S_{21}^2}{(1-S_{11})(1-S_{22})-S_{21}^2}}$$

$$\epsilon^H = n^H z^H$$

$$\mu^H = n^H / z^H$$

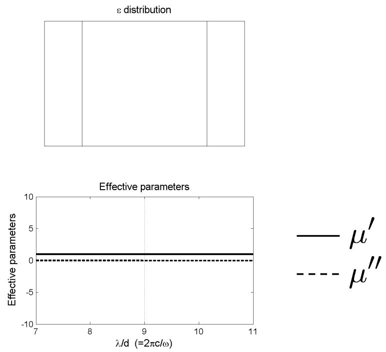
Smith et al. (2005)

2D Electromagnetic Metamaterial design

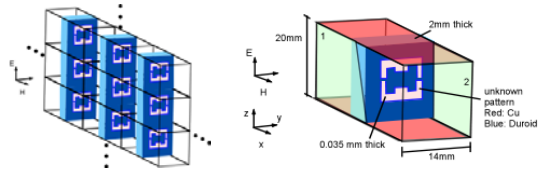


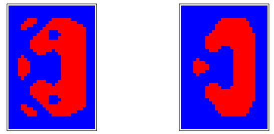
$$\nabla\cdot\left(\frac{1}{\epsilon_r}\nabla H_z\right)+\frac{\omega^2}{c^2}H_z = 0$$

2D design example, min(μ')



First 3D design examples





Diaz and Sigmund (2009), submitted

Conclusions

- Lots of new challenges encountered in applying Topology Optimization to (electromagnetic) wave propagation problems
- Still lots of challenges remaining
  - Homogenization for dynamic problems
  - Coupling to atomistic scale
  - 3D modelling (parallel computing)
- See **www.topopt.dtu.dk** for references and other info

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Ph.D.-course:
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Technical University of Denmark
June 10-16, 2009
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